The Faceless Clock

An old standard algebra problem is the "clock" problem. In typical form it reads, "At what time between one and two o'clock do the two hands of a clock coincide?" When I first taught algebra, I tried to work one of these problems in class with no preparation, and I was still trying to solve it when the bell rang. In some embarrassment I gave several solutions on the next day, including the usual solution with one or two equations to solve and even a solution involving an infinite geometric series. In the end, though, I hit upon an elegant method using nothing but arithmetic. Although this solution surely isn't new, I hadn't seen it before, and perhaps you haven't either.

The key idea is to forget about the face on the clock. Then there is nothing special about the twelve o'clock time when the hands coincide. In twelve hours the hands will coincide at eleven equal intervals. Each interval is thus 12/11 hours, or 1 hour, 5 minutes, 27 3/11 seconds. (This method also works for variations of the clock problem, say, where the hands are to be perpendicular.)

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